

12-3 Probability Assignments

– 8 –

M10/5/MATHL/HP1/ENG/TZ2/XX

7. [Maximum mark: 8]

The function f is defined by $f(x) = e^{x^2 - 2x - 1.5}$.

(a) Find $f'(x)$. [2 marks]

(b) You are given that $y = \frac{f(x)}{x-1}$ has a local minimum at $x = a$, $a > 1$. Find the value of a . [6 marks]

Relative Frequency Theory – probability is the “long-run proportion of repetitions on which an event occurs.”

Probability Rules

- Rule # 1 – The probability of an event occurring is always between 0 and 1.

$$0 \leq P(A) \leq 1$$

- Rule # 2 – The sum of the probabilities of all possible outcomes must = 1. $P(S) = 1$

- Rule #3 – If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities.

$$P(A \cup B) = P(A) + P(B)$$

- Rule #4 – The sum of the probabilities of event A and event A' is 1. $P(A') = 1 - P(A)$

Equally Likely Outcomes – When each possible outcome has the same probability.

Coin: $P(H) = P(T) = 1/2$

Dice: $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$

Probability Calculation for Equally Likely Outcomes using Counting Principles

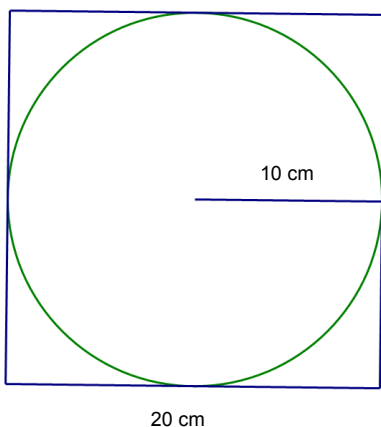
In an experiment where all outcomes are equally, the theoretical probability of an event A is given by:

$$P(A) = \frac{n(A)}{n(S)}$$

Where $n(A)$ is the number of outcomes that make up event A and $n(S)$ is the total number of outcomes in the sample space.

Geometric Probability

Ex1. If a circle with a radius of 10 cm is placed inside a square with a length of 20 cm, what is the probability that a dart, thrown at random will land inside the circle given that it lands inside the square?



$$\begin{aligned} P &= \frac{\text{Area Circle}}{\text{Area of Square}} \\ &= \frac{\pi \cdot 10^2}{20^2} \\ &= \frac{100\pi}{400} = \boxed{\frac{\pi}{4}} \end{aligned}$$

Ex2. The diameter of a the bulls eye is 4 cm. The radius of the middle circle (blue) is 6cm. The radius of the outer circle (pink) is 9 cm. What is the probability that a dart thrown at random will land anywhere in the middle circle but not in the bulls eye given that it hits the target somewhere?

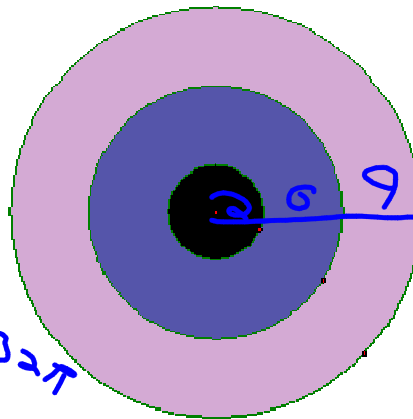
$$81\pi$$

$$36\pi$$

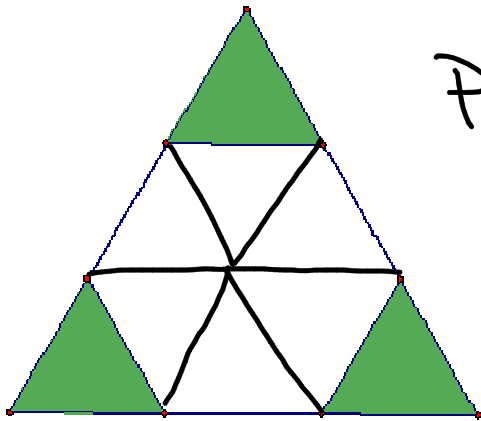
$$4\pi$$

$$36\pi - 4\pi = 32\pi$$

$$P = \frac{32\pi}{81\pi}$$



Ex3. In a recent street fair, students were challenged to hit one of small triangular regions on the large triangular board below with a ping pong ball. Find the probability of hitting a small triangle if the ping pong ball hit large triangular region?

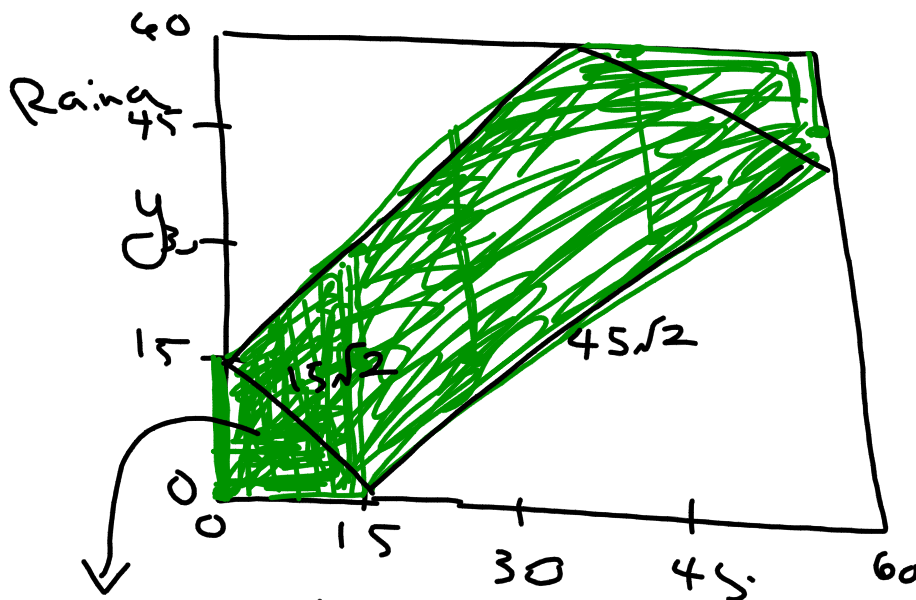


$$P = \frac{3}{9} = \frac{1}{3}$$

Ex4. Lydia and Raina agreed to meet at the museum between 12:00 and 1:00. The first person to arrive will wait for 15 minutes. If the second person does not show up, the first person will leave and they will meet another time. Assuming that their arrivals are at random, what is the probability that they meet?

$x =$ Lydia arrives x min after noon ($0 \leq x \leq 60$)

$y =$ Raina arrives y min after noon ($0 \leq y \leq 60$)



$$\frac{15^2 + 15\sqrt{2} \cdot 45\sqrt{2}}{60^2} = \frac{77}{16}$$

Probabilities Using Permutations and Combinations

Ex5. A squad of 13 players includes 4 brothers. A team of 7 is randomly selected by drawing names from a hat. Determine the probability that the team contains:

- a.) All the brothers
- b.) At least 2 of the brothers

$$\begin{aligned}
 \text{a.) } P &= \frac{\binom{4}{4} \binom{9}{3}}{\binom{13}{7}} \approx .0490 \\
 &= \frac{84}{1716} = \frac{7}{143}
 \end{aligned}$$

$$\begin{aligned}
 \text{b.) } & \text{2 bros or 3 bros or 4 bros} \\
 & \frac{\binom{4}{2} \binom{9}{5} + \binom{4}{3} \binom{9}{4} + \binom{4}{4} \binom{9}{3}}{\binom{13}{7}} \\
 & = \frac{112}{143} \\
 & \approx .783
 \end{aligned}$$

Pg 533 # 1, 3, 6, 8-13, 17, 19-23,
26